

# Spectral continuity in dense QCD

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The vector mesons in three-flavor quark matter with chiral and diquark condensates are studied using the in-medium QCD sum rules. The diquark condensate leads to a mass splitting between the flavor-octet and flavor-singlet channels. At high density, the singlet vector meson disappears from the low-energy spectrum, while the octet vector mesons survive as light excitations with a mass comparable to the fermion gap. A possible connection between the light gluonic modes and the flavor-octet vector mesons at high density is also discussed.

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There are three characteristic phases in quantum chromodynamics (QCD) at finite temperature ( $T$ ) and baryon chemical potential ( $\mu_B$ ): the hadronic phase at low  $T$  and  $\mu_B$ , the quark-gluon plasma phase at high  $T$  and  $\mu_B$  [1], and the color superconductivity (CSC) phase at low  $T$  and high  $\mu_B$  [2]. In recent years, numbers of theoretical and experimental progresses have been made for understanding the thermal (classical) phase transition at high  $T$  between the hadronic phase and the quark-gluon plasma phase. On the other hand, the nonthermal (quantum) phase transition at low  $T$  between the hadronic phase and the CSC phase such as the meson condensed phase, the crystalline Fulde-Ferrell-Larkin-Ovchinnikov phase, gluon condensed phase, etc. [2] has not been fully understood in spite of its relevance to the interior of neutron stars [3].

In determining the phase structure at finite density, interplay between the dynamical breaking of chiral symmetry due to quark–anti-quark pairing,  $\langle \bar{q}q \rangle$ , and color superconductivity due to quark-quark pairing,  $\langle qq \rangle$ , may play a crucial role [4, 5].<sup>1</sup> Recently, it was suggested that such an interplay between  $\langle \bar{q}q \rangle$  and  $\langle qq \rangle$  induced by the QCD axial anomaly leads to a smooth crossover between the hadronic phase and the CSC phase [7]. Furthermore, it was shown in Ref. [8] that the ordinary pion associated with dynamical chiral symmetry breaking at low density and the generalized pion associated with the color superconductivity at high density are mixed with each other in the crossover region: They form a light pionic mode satisfying a generalized Gell-Mann–Oakes–Renner relation in the first order of the quark mass  $m_q$ ;

$$f_\pi^2 m_\pi^2 = m_q (-\langle \bar{q}q \rangle + \Gamma \langle qq \rangle^2). \quad (1)$$

Here  $f_\pi$ ,  $m_\pi$  and  $\Gamma$  are the generalized pion decay constant, the pion mass, and a parameter characterizing the strength of the  $U(1)_A$  axial anomaly, respectively. In

the low density limit with vanishing diquark condensate, Eq. (1) reduces to the usual Gell-Mann–Oakes–Renner relation,  $f_\pi^2 m_\pi^2 = -m_q \langle \bar{q}q \rangle$  [9]. On the other hand, at intermediate to high densities, where the diquark condensate dominates over the chiral condensate, the pion mass may be dictated by the strength of the axial anomaly,  $f_\pi^2 m_\pi^2 \sim m_q \Gamma \langle qq \rangle^2$ . At asymptotic high densities where  $\Gamma$  is suppressed, the  $O(m_q^2)$ -term not considered in the above formula eventually takes over to lead  $f_\pi^2 m_\pi^2 \propto m_q^2 \langle qq \rangle^2$  [10, 11]. This is an explicit realization of the “spectral continuity of hadrons” from low density to high density and may have close relation to the idea of hadron-quark continuity in dense QCD [12].

The purpose of this paper is to present a novel attempt for studying the spectral continuity across the crossover region on the basis of the QCD sum rules [13] and its in-medium generalization [14]. In our approach, the resonance parameters in dense matter are related to the chiral and diquark condensates through gauge invariant correlation functions. In the following, we will focus on the flavor octet and singlet vector mesons (such as  $\rho, \omega, \phi, K^*$ ) in three-flavor quark matter at zero temperature with massless u, d, s quarks. A smooth crossover in the intermediate density region is assumed according to [7, 8].

Let us start with the vector-current correlation function in quark matter:

$$\Pi_{\mu\nu}^{AB}(\omega, \vec{q}) = i \int d^4x e^{iqx} \langle R J_\mu^A(x) J_\nu^B(0) \rangle, \quad (2)$$

where  $R$  denotes the retarded product,  $q^\mu = (\omega, \vec{q})$ ,  $J_\mu^A = \bar{q} \tau^A \gamma_\mu q$  with  $q = (u, d, s)$ . The flavor  $U(3)$  generators  $\tau^A$  ( $A = 0, \dots, 8$ ) are normalized as  $\text{tr}[\tau^A \tau^B] = 2\delta^{AB}$ . The expectation value of an operator  $\mathcal{O}$  with respect to the quark matter is denoted as  $\langle \mathcal{O} \rangle$ . One can decompose Eq.(2) by introducing the longitudinal and transverse invariants ( $\Pi_{L,T}^{AB}$ ) as  $\Pi_{00}^{AB} = \vec{q}^2 \Pi_L^{AB}$  and  $\Pi_{ij}^{AB} = (\delta_{ij} - q_i q_j / \vec{q}^2) \Pi_T^{AB} + q_i q_j \omega^2 \Pi_L^{AB} / \vec{q}^2$ . Since there is no distinction between the transverse and longitudinal components in the limit  $\vec{q} = 0$ , it is sufficient to consider  $\Pi_L^{AB}(\omega) \equiv \Pi_L^{AB}(\omega, \vec{0})$  in treating the vector mesons at rest in quark matter [14]. Also, it is convenient to define the flavor-octet and singlet correlation functions because

<sup>1</sup> Another important interplay would be induced by confinement characterized by the Polyakov loop [6] which we will not discuss in this paper.

of the flavor  $SU(3)$  symmetry:  $\Pi_L^{(8)} \equiv \frac{1}{8} \sum_{A=1}^8 \Pi_L^{AA}$  and  $\Pi_L^{(1)} \equiv \Pi_L^{00}$ .

The dispersion relation for  $\Pi_L$  reads

$$\Pi_L(\omega) = \int_0^\infty \frac{\rho(u)}{u^2 - (\omega + i\epsilon)^2} du^2 - (\text{subtraction}), \quad (3)$$

where the spectral function is given by  $\rho(u) = (1/\pi)\text{Im}\Pi_L(u)$ . In QCD sum rules,  $\Pi_L(\omega)$  in the deep Euclidean region ( $Q^2 \equiv -\omega^2 \rightarrow \infty$ ) is evaluated with the operator product expansion (OPE) and is compared with the dispersion integral in Eq. (3) under suitable parametrization of  $\rho(u)$ . Not only the Lorentz scalar operators but also the tensor operators in OPE contribute to the correlation functions in the medium [14]. In the massless quark matter with finite quark chemical potential  $\mu$ , the matrix element of an operator with canonical dimension  $d$  has the general form

$$\langle \mathcal{O} \rangle = f(\alpha_s) \mu^d + \langle \langle \mathcal{O} \rangle \rangle, \quad (4)$$

where the first term is calculable order by order in  $\alpha_s$ , while the second term contains genuine nonperturbative effects such as the chiral and diquark condensates.

To show how our approach works, we start with noninteracting quark matter with the quark chemical potential  $\mu$ . The OPE for  $\Pi_L^{(8,1)}$  up to  $O(1/Q^6)$  in this case reads [14]

$$\Pi_L^{(\text{free})}(Q) = -\frac{1}{2\pi^2} \ln Q^2 + \frac{16}{9} \frac{\langle q^\dagger i \partial_0 q \rangle}{Q^4} + \frac{64}{9} \frac{\langle q^\dagger i \partial_0^3 q \rangle}{Q^6}. \quad (5)$$

The matrix elements of the twist 2 quark operators in Eq. (5) can be evaluated as

$$\langle q^\dagger i \partial_0^n q \rangle = (-i)^{n-1} \frac{9\mu^{n+3}}{(n+3)\pi^2}. \quad (6)$$

The asymptotic expansion Eq. (5) matches exactly the spectral function in the free quark matter as shown in Fig. 1(a):

$$\rho^{(\text{free})}(u) = F\delta(u^2) + A\theta(u^2 - S_0), \quad (7)$$

with the resonance parameters,  $S_0 = (2\mu)^2$ ,  $A = 1/(2\pi^2)$ , and  $F = AS_0$ . The pole part at zero energy corresponds to the scattering of quarks on the Fermi surface with the external current, i.e., the Landau-damping term. The continuum part corresponds to the decay of the external current into quark-antiquark pair with the Pauli blocking effect. The corrections of the form  $\alpha_s \ln Q^2$  and  $\alpha_s(\mu/Q)^n$  to Eq. (5) can be taken into account in perturbation theory and are compensated by the perturbative corrections to the resonance parameters and the spectral shape in Eq. (7).

Let us now study the effect of “nonperturbative” condensates. They introduce extra four-quark terms in

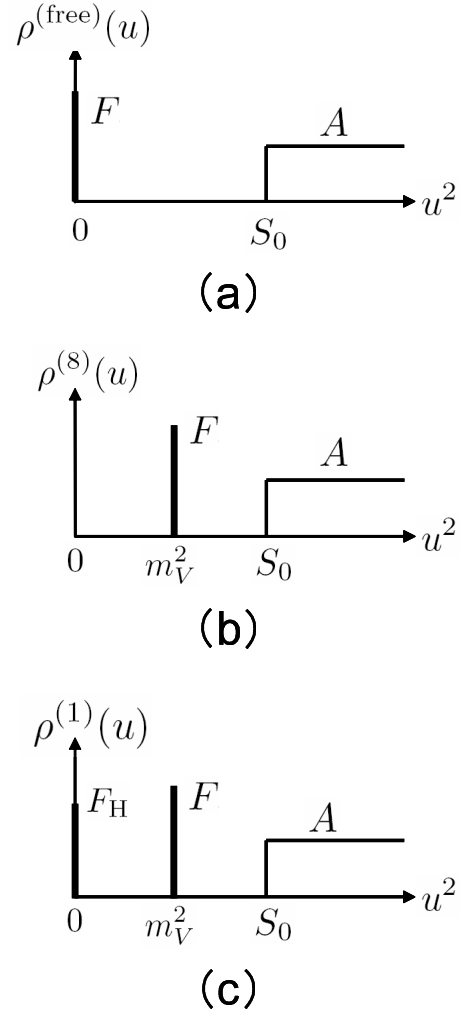


FIG. 1: The spectral function for the correlation of vector currents in dense quark matter: (a) The case without nonperturbative condensates, (b) the case for octet correlation with chiral and diquark condensates, and (c) the case for singlet correlation with chiral and diquark condensates.

Eq. (5) [14, 15]

$$\Pi_L^{(8,1)} = \Pi_L^{(\text{free})} + \delta\Pi_L^{(8,1)}, \quad (8)$$

$$\delta\Pi_L^{(8)} = -\frac{\pi\alpha_s}{Q^6} \left[ \frac{1}{4} \langle\langle (\bar{q}\gamma_\mu\gamma_5\tau^a\lambda^{a'}q)^2 \rangle\rangle + \frac{8}{27} \langle\langle (\bar{q}\gamma_\mu\lambda^{a'}q)^2 \rangle\rangle \right], \quad (9)$$

$$\delta\Pi_L^{(1)} = -\frac{\pi\alpha_s}{Q^6} \left[ 2 \langle\langle (\bar{q}\gamma_\mu\gamma_5\tau^0\lambda^{a'}q)^2 \rangle\rangle + \frac{8}{27} \langle\langle (\bar{q}\gamma_\mu\lambda^{a'}q)^2 \rangle\rangle \right], \quad (10)$$

where  $\tau^{a=1\sim 8}$  ( $\lambda^{a'=1\sim 8}$ ) are the flavor (color)  $SU(3)$  generators, and the repeated indices are summed over from 1 to 8. The four-quark condensates in Eqs. (9) and (10) induce two independent corrections to the current correlation function as illustrated in Fig. 2 for  $\langle\langle (\bar{q}\gamma_\mu\gamma_5\tau^A\lambda^{a'}q)^2 \rangle\rangle$ . The graph in Fig. 2(a) corresponds to  $\bar{q}q$  pairing which connects the vector currents with opposite chiralities, while that in Fig. 2(b) corresponds to  $qq$  pairing which connects the vector currents with the same

chirality. Here we consider the most attractive channels in each case, namely  $\bar{q}q$  pairing in the color-flavor singlet and spin-parity  $0^+$  channel, and  $qq$  pairing in the color-flavor locked (CFL) and spin-parity  $0^+$  channel [10],

$$\langle\langle \bar{q}_i^\alpha q_j^\alpha \rangle\rangle = \text{diag}(\sigma, \sigma, \sigma), \quad (11)$$

$$\frac{1}{4}\epsilon_{ijk}\epsilon_{\alpha\beta\gamma}\langle\langle q_j^\beta C\gamma_5\Lambda_+ q_k^\gamma \rangle\rangle = \text{diag}(\varphi, \varphi, \varphi). \quad (12)$$

Here  $i, j, k$  ( $\alpha, \beta, \gamma$ ) are the flavor (color) indices and  $\Lambda_+$  is the projection operator to the positive energy quarks. After rewriting the four-quark operators in Eqs. (9) and (10) in the chiral basis and making the appropriate Fierz rearrangement together with the factorization ansatz [16]

$$\langle\langle \mathcal{O} \rangle\rangle = \sum_l \langle\langle \mathcal{P}_l \cdot \mathcal{P}_l \rangle\rangle \simeq \sum_l \langle\langle \mathcal{P}_l \rangle\rangle^2, \quad (13)$$

we obtain

$$\delta\Pi_L^{(8,1)} \simeq \Pi_\sigma + \Pi_\varphi^{(8,1)}, \quad (14)$$

$$\Pi_\sigma = -\frac{448\pi\alpha_s}{81Q^6}\sigma^2, \quad (15)$$

$$\Pi_\varphi^{(8)} = -\frac{5}{22}\Pi_\varphi^{(1)} = -\frac{320\pi\alpha_s}{27Q^6}\varphi^2. \quad (16)$$

Since the chiral condensate is flavor-diagonal, it does not distinguish between octet and singlet. On the other hand, the diquark condensate has color-flavor structure which can differentiate the flavor structure in Eqs. (9) and (10). This is the reason why the flavor-octet and flavor-singlet vector mesons, which are almost degenerate at low density, tend to split at high density as will be shown below.

Let us first examine the flavor-octet vector mesons under the influence of the nonperturbative condensates. Since there is a gap at the Fermi surface in the color superconducting quark matter, we do not expect the Landau-damping contribution, in contrast to Eq. (7). Instead we may have flavor-octet collective modes with finite mass. With these observations, we introduce the following ansatz for the spectral function as shown in Fig. 1(b):

$$\rho^{(8)}(u) = F\delta(u^2 - m_V^2) + A\theta(u^2 - S_0). \quad (17)$$

Here  $m_V$  and  $S_0$  are the vector meson mass and the continuum threshold, respectively.

Substituting the ansatz Eq. (17) into Eq. (3), carrying out the asymptotic expansion in terms of  $1/Q^2$  after subtraction, and comparing the result with the OPE expression in Eq. (8), we end up with the following finite energy sum rules:

$$F - AS_0 = 0, \quad (18)$$

$$2Fm_V^2 - AS_0^2 = -\frac{(2\mu)^4}{2\pi^2}, \quad (19)$$

$$3Fm_V^4 - AS_0^3 = -\frac{(2\mu)^6}{2\pi^2} + \langle\langle \mathcal{O}^{(8)} \rangle\rangle, \quad (20)$$

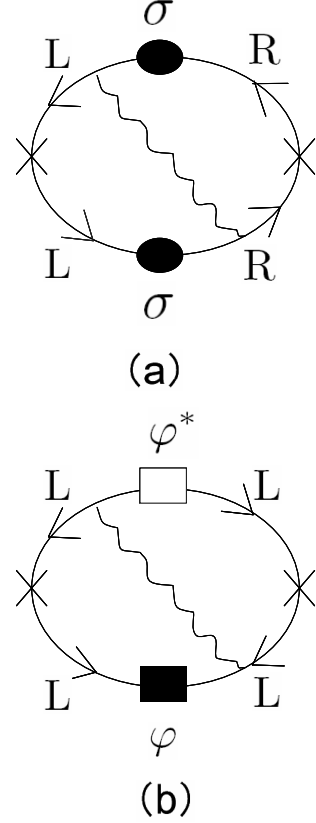


FIG. 2: Examples of  $\delta\Pi_L^{(8,1)}$  from (a) the chiral condensate and (b) the diquark condensate. Quarks are denoted by solid lines with their chirality structure (L and R), while the gluon is denoted by a wavy line. the chiral condensate  $\sigma$  by a black circle and the diquark condensate  $\varphi$  ( $\varphi^*$ ) by a black (white) square.

with  $A = 1/(2\pi^2)$  and

$$\langle\langle \mathcal{O}^{(8)} \rangle\rangle = -\frac{448}{27}\pi\alpha_s \left( \sigma^2 + \frac{15}{7}\varphi^2 \right). \quad (21)$$

When  $\mu = 0$  and  $\varphi = 0$ , the solution of these equations reduces to the standard formula [17]:

$$\left( m_V^{(8)} \right)^2 \xrightarrow{\mu \rightarrow 0} \left( \frac{448\pi^3\alpha_s}{27}\sigma^2 \right)^{\frac{1}{3}}. \quad (22)$$

When  $\mu \neq 0$  with both  $\sigma$  and  $\varphi$  finite, we obtain the quartic equation for  $S_0$  from Eqs. (18-20),

$$t^4 + 6t^2 - 4(1 + r^{(8)})t - 3 = 0, \quad (23)$$

where  $t$  and  $r^{(8)}$  are dimensionless parameters defined as  $t = S_0/(2\mu)^2$  and  $r^{(8)} \equiv -2\pi^2\langle\langle \mathcal{O}^{(8)} \rangle\rangle/(2\mu)^6$ , respectively. In a situation where  $0 \leq r^{(8)} \ll 1$ , we have a

unique solution,

$$\left(m_V^{(8)}\right)^2 \simeq \frac{56\pi^3\alpha_s}{81\mu^4} \left(\sigma^2 + \frac{15}{7}\varphi^2\right), \quad (24)$$

$$S_0 \simeq (2\mu)^2 + (m_V^{(8)})^2, \quad (25)$$

$$F = \frac{S_0}{2\pi^2}. \quad (26)$$

This is a new formula relating the mass of octet vector mesons to the chiral and diquark condensates. Since  $0 \leq r^{(8)} < 0.1$  is satisfied as long as  $(|\sigma|^{1/3}, |\varphi|^{1/3})/\mu < 0.3/\alpha_s^{1/6}$ , the small  $r^{(8)}$  approximation is safe for all densities expected in quark matter. The characteristic value of the flavor-octet vector meson mass is  $m_V^{(8)} \simeq 100\text{MeV}$  for  $\mu = 500\text{ MeV}$ ,  $\alpha_s \sim 1$  and  $\sigma \sim \varphi \sim (150\text{MeV})^3$ .

At asymptotic high density, we have  $\sigma \sim 0$  and the weak coupling relation [18],

$$\varphi = \frac{3\mu^2\Delta}{\pi\sqrt{2\pi\alpha_s}}, \quad (27)$$

with the fermion gap  $\Delta$ . Then we obtain

$$m_V^{(8)} \rightarrow \sqrt{\frac{20}{3}} \Delta, \quad (28)$$

which implies the existence of light vector mesons in the flavor-octet channel in the high density limit [19]. A similar result was also obtained in [20, 21].

The above discussion leads to a question about the fate of the flavor-singlet vector meson. In this channel, unlike the vector octet, the Nambu-Goldstone scalar (referred to as H) associated with the spontaneous breaking of  $U(1)_B$  symmetry contributes to the spectral function [22]. (This is analogous to the situation where not only the  $a_1$  meson but also the pion contribute to the axial-vector current correlation in the vacuum.) Then we should modify the ansatz for the spectral function Eq. (17) as

$$\rho^{(1)}(u) = F_H\delta(u^2) + F\delta(u^2 - m_V^2) + A\theta(u^2 - S_0), \quad (29)$$

This situation is shown in Fig. 1(c). The finite energy sum rules in the singlet channel are obtained by the replacements:  $F \rightarrow F + F_H$  in Eq. (18) and

$$\langle\langle \mathcal{O}^{(8)} \rangle\rangle \rightarrow \langle\langle \mathcal{O}^{(1)} \rangle\rangle = -\frac{448}{27}\pi\alpha_s \left(\sigma^2 - \frac{66}{7}\varphi^2\right) \quad (30)$$

in Eq. (20). Although the sum rules are not closed due to the extra parameter  $F_H$ , we can still select a possible solution under physical constraints such as the positivity of the spectral function and the small magnitude of  $r^{(1)} \equiv -2\pi^2\langle\langle \mathcal{O}^{(1)} \rangle\rangle/(2\mu)^6$ . It is remarkable here that unlike the case of  $r^{(8)}$ , the parameter  $r^{(1)}$  could change its sign depending on the relative magnitude of the condensates  $\sigma$  and  $\varphi$ .

For  $r^{(1)} > 0$  with fixed  $F$ , we find two solutions: a heavy one with  $S_0 > (m_V^{(1)})^2 > 2\mu^2$  and a light one,

$$\left(m_V^{(1)}\right)^2 \simeq \frac{1}{f} \frac{56\pi^3\alpha_s}{81\mu^4} \left(\sigma^2 - \frac{66}{7}\varphi^2\right), \quad (31)$$

with  $f \equiv 2\pi^2 F/(2\mu)^2$ . Since the mass in Eq. (31) is essentially determined by the QCD condensates, we adopt this light solution as the physical one connected continuously to the flavor-singlet vector meson at low density.

For  $r^{(1)} < 0$  with fixed  $F$ , we have only a heavy solution with  $S_0 > (m_V^{(1)})^2 > (2\mu)^2$ . Since the mass in this case is above the threshold for  $\bar{q}q$  decay, we do not expect it to appear as a sharp resonance in reality. This indicates that the light flavor-singlet vector meson can exist only below a critical chemical potential  $\mu_c$  determined by the condition,  $\sigma^2 = \frac{66}{7}\varphi^2$ . The value of  $\mu_c$  cannot be estimated without knowing the precise  $\mu$  dependence of  $\sigma$  and  $\varphi$  which is beyond the ability of the QCD sum rules. However, as long as  $|\sigma|$  ( $|\varphi|$ ) is a decreasing (increasing) function of the chemical potential,  $\mu_c$  is located below the crossover point at  $|\sigma| = |\varphi|$ .

Note here that, for sufficiently high density with  $|r^{(1)}| \ll 1$  with fixed  $F$ , one can show a relation

$$F_H \rightarrow \frac{2\mu^2}{\pi^2} \quad (32)$$

from our sum rules. This is consistent with the known  $F_H$  estimated in the weak coupling calculation at high density [11, 22].

On the basis of our analysis, we draw a schematic plot of the masses of the octet and singlet vector mesons as a function of the chemical potential  $\mu$  in Fig. 3. At low density, vector mesons are almost degenerate and form a nonet structure. As the density increases, the effect of the diquark condensate becomes prominent: It contributes positively (negatively) to  $m_V^{(8)}$  ( $m_V^{(1)}$ ) as seen from Eqs. (24) and (31), and increases their mass splitting. At higher density, the octet vector mesons survive as light excitations, while the singlet vector meson disappears from the low-energy spectrum.

Remembering the fact that flavors and colors are mixed in the CFL phase, flavor-octet vector mesons at high density cannot be distinguished from the color-octet gluons. This suggests that the light gluonic modes with the mass of  $O(\Delta)$  (the light CFL plasmon) found in [23] has a close connection to our octet vector mesons at asymptotic high density.

In summary, we have studied the vector mesons in quark matter with massless u, d, s quarks by using the in-medium QCD sum rules. We have derived formulas relating vector meson masses to the chiral and diquark condensates. Owing to the different role played by the chiral and diquark condensates, we find spectral continuity (discontinuity) in the flavor-octet (flavor-singlet)

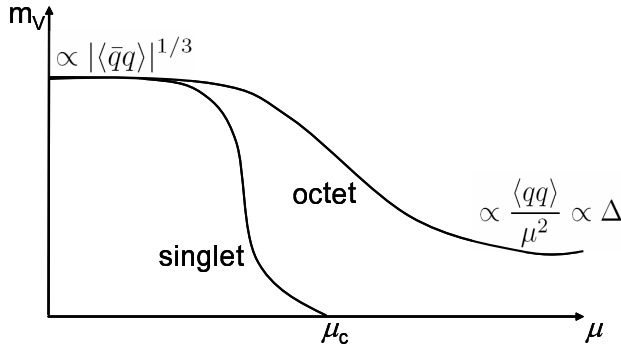


FIG. 3: Schematic plot of the masses of flavor-octet and flavor-singlet vector mesons as a function of  $\mu$ . The magnitude of the chiral condensate  $\sigma$  (the diquark condensate  $\varphi$ ) is assumed to decrease (increase) monotonically as  $\mu$  increases.

channel. In particular, the octet vector mesons survive at high density as light excitations with a mass comparable to the fermion gap.

Our sum-rule approach provides a novel tool to analyze not only the vector mesons but also the other hadrons in dense matter. For example, the spectral continuity of flavored baryons at low density and colored quarks at high density is one of the interesting problems to be examined. Also, it is important to study how the strange quark mass whose main effect in OPE comes from the operator  $m_s \bar{s}s$ , affects the spectral continuity found in massless quark matter.

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